

Kerr Naked Singularities as Particle Accelerators

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Abstract. We investigate here the particle acceleration by Kerr naked singularities. We consider a collision between particles dropped in from infinity at rest, which follow geodesic motion in the equatorial plane, with their angular momenta in an appropriate finite range of values. When an event horizon is absent, an initially infalling particle turns back as an outgoing particle, when it has the angular momentum in an appropriate range of values, which then collides with infalling particles. When the collision takes place close to what would have been the event horizon in the extremal case, the center of mass energy of collision is arbitrarily large, depending on how close is the overspinning Kerr geometry to the extremal case. Thus the fast rotating Kerr configurations if they exist in nature could provide an excellent cosmic laboratory to probe ultra-high-energy physics.

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1. Introduction

An intriguing possibility to study new physics at ultrahigh energies, which remains unexplored by terrestrial particle accelerators, is to make use of naturally occurring astrophysical exotic objects. In this spirit, the divergence of center of mass energy of infalling particles colliding near the event horizon of near extremal Kerr blackholes, and the observational signatures of such a process in the context of dark matter annihilations were studied recently [1] [2]. However, this process suffers from several drawbacks, such as the extreme fine-tuning required of the angular momentum of the particles, and also an infinite proper time needed for the collision events to take place [16].

In this note we investigate the particle collision with ultrahigh energies in the background of near extremal Kerr naked singularities, transcending the Kerr bound by vanishingly small amount. The interesting point that we show here is, in such a process the drawbacks mentioned above are naturally circumvented, due to the absence of an event horizon. This allows us to consider high energy collisions between ingoing and outgoing particles in a generic manner, unlike in the blackhole case.

2. Geodesics in Kerr geometry in equatorial plane

We examine here the particle collisions in the background of a Kerr naked singularity. For simplicity and clarity we focus on the test particles following timelike geodesics in the equatorial plane. The Kerr metric [3] in Boyer-Lindquist coordinates (t, r, θ, ϕ) in the equatorial plane, $\theta = \frac{\pi}{2}$ is given by

$$ds^2 = -\left(1 - \frac{2}{r}\right) dt^2 - \frac{4a}{r} dt d\phi + \left(\frac{r^2}{\Delta}\right) dr^2 + r^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2}{r}\right) d\phi^2 \quad (1)$$

where $\Delta = r^2 + a^2 - 2r$. We work in the units $c = G = M = 1$, M being the mass and a the angular momentum parameter. The event horizon is obtained by solving $\Delta = 0$. It then follows that when $a > 1$, there is no event horizon and the timelike naked singularity at $r = 0, \theta = \frac{\pi}{2}$ is exposed to asymptotic observers. We also note that for extremal Kerr blackhole, namely with $a = M = 1$, the event horizon is located at $r = 1$.

The metric admits killing vectors $\partial_t, \partial_\phi$ and thus the quantities

$$\begin{aligned} E &= -g_{\mu\nu} (\partial_t)^\mu U^\nu = -g_{tt} U^t - g_{t\phi} U^\phi \\ L &= g_{\mu\nu} (\partial_\phi)^\mu U^\nu = -g_{\phi t} U^t - g_{\phi\phi} U^\phi \end{aligned}$$

are conserved along the geodesics. These are interpreted as conserved energy and angular momentum per unit mass of the particle, U being the four velocity of the particle. Solving for U^t, U^ϕ we get, [1],[4],

$$U^t = \frac{1}{\Delta} \left[\left(r^2 + a^2 + \frac{2a^2}{r} \right) E - \frac{2a}{r} L \right] = \frac{1}{r^2} \left[-a(aE - L) + \frac{r^2 + a^2}{\Delta} T \right] \quad (2)$$

$$U^\phi = \frac{1}{\Delta} \left[\left(1 - \frac{2}{r} \right) L + \frac{2a}{r} E \right] = \frac{1}{r^2} \left[(L - aE) + \frac{a}{\Delta} T \right] \quad (3)$$

where $T = E(r^2 + a^2) - La$.

From (2),(3), $U^\theta = 0$ and normalization condition $U^\mu U_\mu = -1$ for a timelike geodesic, the radial component of velocity can be written as,

$$\begin{aligned} U^r &= \pm \sqrt{E^2 - 1 + \frac{2}{r} - \frac{(L^2 - a^2(E^2 - 1))}{r^2} + \frac{2(L - aE)^2}{r^3}} \\ &= \pm \frac{1}{r^2} \sqrt{T^2 - \Delta(r^2 + (L - aE)^2)} \end{aligned} \quad (4)$$

Here \pm stand for radially outgoing and ingoing geodesics respectively. The above equation can be cast in the form

$$\begin{aligned} U^{r^2} + V_{eff}(L, E, r) &= 0 \\ V_{eff} &= -E^2 + 1 - \frac{2}{r} + \frac{(L^2 - a^2(E^2 - 1))}{r^2} - \frac{2(L - aE)^2}{r^3} \end{aligned} \quad (5)$$

where $V_{eff}(L, E, r)$ can be thought of as an effective potential for the radial motion. The center of mass energy of collision [1] of two particles with velocities U_1 and U_2 is given by

$$E_{c.m.}^2 = 2m^2(1 - g_{\mu\nu}U_1^\mu U_2^\nu) \quad (6)$$

We restrict our attention here to the geodesics with conserved energy per unit mass $E = 1$, corresponding to the case of marginally bound particles, released at infinity from rest, whose energy comes solely from the gravitational acceleration of the Kerr spacetime.

3. Particle acceleration for extremal Kerr blackholes

Towards considering the particle collisions in the Kerr geometry, we first note that in the Banados-Silk-West (BSW) mechanism of particle acceleration by near-extremal Kerr blackholes, two identical particles at rest each with mass m are released from infinity, and are made to collide near the horizon of the Kerr black hole. These ingoing particles are highly blue-shifted by the time they reach the event horizon. But in most of the cases they reach the horizon almost perpendicular to it, so the relative velocity of approach of two particles happens to be small. Therefore the center of mass energy of particles is finite and not significantly larger than their rest mass energy. Thus particles which participate in collisions must have large and opposite angular momenta, so as to maximize the relative velocity of collision between the particles. Particles with small angular momentum fall into the blackhole almost perpendicular to the horizon, whereas particles with rather large angular momenta turn back even before they could reach the horizon. It follows that one must then fine-tune the angular momentum of a particle to a largest possible value that still makes it possible for it to reach the event horizon. The center of mass energy of collision is then maximized in that case.

When the black hole is close to extremality, this fine-tuned angular momentum approaches a value $L = \Omega_H^{-1}$, where Ω_H is the angular velocity of the event horizon, and

the following condition is also satisfied,

$$V_{eff} = \frac{dV_{eff}}{dr} = 0 \quad (7)$$

This essentially implies that the particle travels almost parallel to the event horizon, which is a null surface, and thus it is ultra-relativistic with respect to the other particle with which it collides. This leads to the divergence of the center of mass energy in the BSW mechanism. Here the equation (7) implies that the proper time required for the particle to reach the horizon and also for such a collision to take place approaches infinity.

4. Particle acceleration by Kerr naked singularities

The origin of such issues in the case of blackholes is that the event horizon is a one way membrane. The chosen location for collisions with divergent center of mass energy has to be arbitrarily close to the event horizon, because it is an infinite blueshift surface for the particles approaching it. When two infinitely blueshifted particles collide near the event horizon of the blackhole with sufficiently large relative velocities, the center of mass energy of collision is bound to diverge. In case of a Kerr black hole, it is not possible to have a collision between the ingoing and outgoing particles due to the absence of the outgoing particles near the event horizon which is a one way membrane in the spacetime. Therefore one must consider collision between only the infalling particles near the horizon towards the purpose of high energy collisions. In such a case, the only way to maximize relative velocity between them is to fine-tune the angular momentum of one of the particles.

Such a problem is naturally circumvented if we consider a near-extremal Kerr naked singularity, rather than a near-extremal Kerr blackhole. In such a case there is no event horizon existing in the spacetime, thus allowing for the possibility of collision between an *ingoing* and another *outgoing* particles, as shown in Fig1. Then the relative velocity of collision between these two particles can be very large, and the requirement of fine-tuning of angular momentum and various issues arising from it disappear. As we show later in this section, the range of allowed particle angular momenta is a finite interval, unlike the single fine-tuned value in the blackhole case.

Since the naked singularity is assumed to be near extremal with

$$a - M = a - 1 = \epsilon \rightarrow 0, \quad (8)$$

the surface $r = 1$, which would have been the event horizon for the extremal blackhole, is still a surface with arbitrarily large blueshift for the particles approaching it. This follows from the fact that

$$\Delta(r = 1) \approx 2\epsilon \rightarrow 0 \quad (9)$$

Therefore, the center of mass energy of collision between the two particles, which approach each other from the opposite directions with large relative velocity and that suffer from extremely large blueshift as they approach $r = 1$, can be arbitrarily large.

Let us now consider a collision between two identical particles of mass m , which follow a geodesic motion along the equatorial plane. The particles are assumed to be at rest at infinity, so the conserved energy of each particle is $E = 1$. The effective potential (5) for the radial motion in that case is given by,

$$V_{eff} = -\frac{2}{r} + \frac{L^2}{r^2} - \frac{2(L-a)^2}{r^3} \quad (10)$$

For a particle with the orbital angular momentum $L = 0$, the above expression for the effective potential implies that the gravity is always attractive in the equatorial plane (see Fig2). This is unlike the case where the gravity is repulsive in Kerr geometry, off the equatorial plane, in the vicinity of naked singularity [5]. Thus such a particle will fall in with an ever-increasing radial component of velocity and eventually hit the naked singularity at $r = 0, \theta = \frac{\pi}{2}$. It then follows that if the ingoing particle were to turn back, it must have necessarily a non-zero angular momentum.

An initially ingoing particle will turn back if its effective potential for radial motion admits a zero. The radial coordinate where the particle undergoes a reflection is the larger root of the equation

$$V_{eff}(r) = 0 \quad (11)$$

which is given by,

$$r = r_{refl} = \frac{L^2}{4} [1 + \sqrt{D}] \quad (12)$$

where $D = 1 - 16\frac{(L-a)^2}{L^4}$. For the existence of a real root of the equation (11) above, we must have $D > 0$.

It can be easily seen that for extremely small values of the angular momentum $|L| \rightarrow 0$, $D \rightarrow -\infty$, the equation (12) does not admit any real roots. Thus in such a case the ingoing particle never turns back and it continues its motion inwards to hit the singularity.

On the other hand, for the very large values of the angular momenta, as $|L| \rightarrow \infty$, we have $D \rightarrow 1$. Therefore in that case the ingoing particle gets reflected at an extremely large value of the radial coordinate $r_{refl} \approx \frac{L^2}{2}$.

In fact, there exists an intermediate critical value of the angular momentum L for the particle, which is given by a solution of the equation $D = 0$. This has the property that if the angular momentum is larger than this critical value, the initially infalling particle eventually turns back, and if the angular momentum is smaller than this value, then the particle will fall inwards and would eventually hit the singularity.

The angular momentum of the particle can be oriented either parallel to the spin of a naked singularity or it could be antiparallel. We first assume that it is parallel so that $L > 0$.

The equation $D = 0$ is to be solved in order to obtain a minimum critical value of the angular momentum L_{crit} for the particle to turn back. We can write this equation as,

$$L^2 = 2|L - a| = \pm 2(a - L) \quad (13)$$

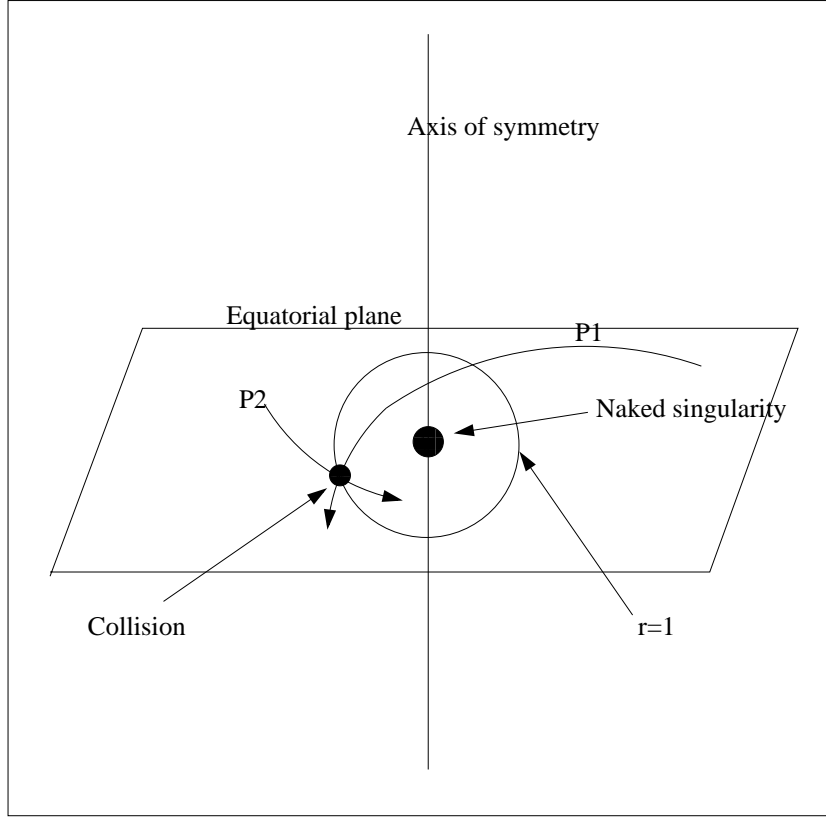


Figure 1. Schematic diagram of a Kerr spacetime with naked singularity. One of the particles which is initially ingoing turns back due to the angular momentum barrier, and it then collides with another ingoing particle near $r = 1$. Both the particles follow the geodesic motion in the equatorial plane. The center of mass energy of collision is arbitrarily large in the limit $a \rightarrow 1^+$

The positive or negative signs in the equation above stand for the cases where the critical angular momentum is smaller or larger than the Kerr spin parameter a respectively.

We note that in the Kerr blackhole case, if we write the equation (13) above with a positive sign and obtain the critical angular momentum from the same as a solution, then if the ingoing particle were to turn back, the turning point happens to be necessarily inside the horizon. Such a scenario is clearly not allowed. This then implies that in the blackhole case, for the particle to turn back, the allowed values for the critical angular momentum must be larger than the Kerr spin parameter. In that case we have to solve the above equation with a minus sign. Then the angular momentum that solves (13) yields a legitimate turning point which is outside the event horizon. Thus in the blackhole case, the solution to (13) with a minus sign is the critical angular momentum for the particle to turn back.

The situation is quite different for a naked singularity, which corresponds to $a > 1$ values, essentially due to the absence of an event horizon. The legitimate turning point for an ingoing particle in this case could be in principle all the way upto the singularity, which is located at $r = 0, \theta = \frac{\pi}{2}$. This is unlike the blackhole case, where the turning

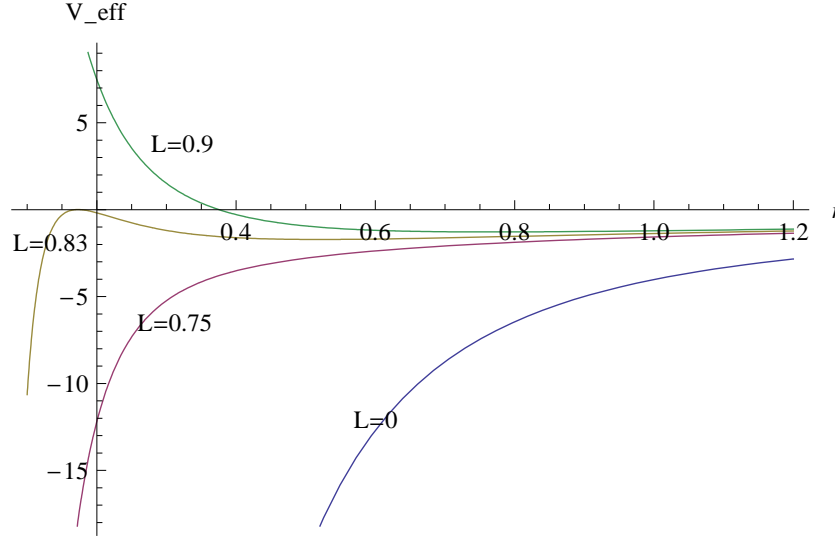


Figure 2. The effective potential is plotted for a particle with $E = 1$. The Kerr parameter is assumed to be $a = 1.005$. The effective potential is a monotonically decreasing function for $L = 0$, indicating that the particle will travel radially inwards with ever-increasing radial velocity component and hit the singularity. The behavior is similar for the particle with subcritical angular momentum $L = 0.75$. The effective potential with the critical angular momentum $L = 0.83$ barely manages to touch the zero at $r = 0.18$. This is the minimum value of the angular momentum of the particle for which it turns back. The effective potential admits a zero at $r = 0.37$, where the particle turns back, for a supercritical value of the angular momentum $L = 0.9$.

point must be strictly located outside the horizon as we noted above. Since we are looking for the smallest value of the angular momentum for which an initially ingoing particle turns back, we solve (13) with a positive sign. It turns out, as we show below, that the turning point, with the critical angular momentum obtained by solving (13) with the positive sign, is at a positive value of the radial coordinate. Thus, the allowed critical value of the angular momentum for the particle to turn back happens to be smaller than the Kerr spin parameter in this case.

We solve (13) to obtain

$$L_{crit} = 2 \left(-1 + \sqrt{1 + a} \right) \quad (14)$$

The turning point for the ingoing particle with the critical angular momentum L_{crit} , will be

$$r_{refl,crit} = \frac{L_{crit}^2}{4} = \left(\sqrt{1 + a} - 1 \right)^2 \quad (15)$$

For Kerr naked singularities, since $a > 1$, we get a turning point at a location away from the naked singularity at $r = 0, \theta = \frac{\pi}{2}$.

The fact that the solution to (13), with a plus sign indeed yields a critical angular momentum, which happens to be smaller than the Kerr spin, is explicitly demonstrated by Fig2. The behavior of the effective potential for the particles with angular momenta smaller, larger and equal to this critical value are also plotted in Fig2. The effective

potential for the subcritical angular momentum does not admit a zero, indicating the absence of any turning point. The effective potential for the critical value of the angular momentum barely manages to take a zero value. It in fact admits a maximum. The effective potential for the particle with supercritical angular momentum cuts the horizontal axis, from where the ingoing particle can turn back.

We clarify that the turning point $r_{refl,crit}$ for a particle with the critical angular momentum has the following sense. The ingoing particle will asymptotically approach $r = r_{refl,crit}$ as the proper time tends to infinity, since both the effective potential and its derivative vanish as it can be seen clearly in Fig2. However, for any value of the angular momentum larger than this critical value, the particle turns back. This is because the effective potential is zero but its derivative takes a nonzero value at the turning point.

It follows that the angular momentum of the particle should be strictly larger than the critical value if it is to turn back, and we have,

$$L > 2 \left(-1 + \sqrt{1 + a} \right) \quad (16)$$

We note that the angular momentum of the particle to turn back can either be smaller or larger than the Kerr spin parameter a as long as it satisfies the condition given above. It is only the critical value of the angular momentum of the particle which is smaller than the Kerr spin parameter.

Since we want collisions to take place at $r = 1$, one of the colliding particles must get reflected back from a radial coordinate $r < 1$. Thus we further impose a condition that

$$r_{refl} < 1 \quad (17)$$

The upper limit on the angular momentum of the particle obtained from the equation above is given by

$$L < \left(2a - \sqrt{2a^2 - 2} \right) \quad (18)$$

The effective potential for the angular momenta smaller, larger and equal to the above value are plotted in Fig3. As is evident from the figure, only the effective potential of the particles with angular momenta satisfying the conditions above admits a zero for $r < 1$.

Combining together the conditions (16) and (18), we obtain the interval of the allowed angular momenta values of one of the particles which is initially ingoing and later turns back, as below,

$$2 \left(-1 + \sqrt{1 + a} \right) < L < \left(2a - \sqrt{2a^2 - 2} \right) \quad (19)$$

The only condition that must be imposed on the second particle is (18), so that it does not get reflected back at $r > 1$ and actually reaches $r = 1$ as an ingoing particle.

Therefore, the particle dropped in from infinity, which moves along the equatorial plane with angular momentum in the range given by (19), crosses $r = 1$ as an ingoing particle, and it is then reflected back at the radial coordinate $r < 1$. It then again

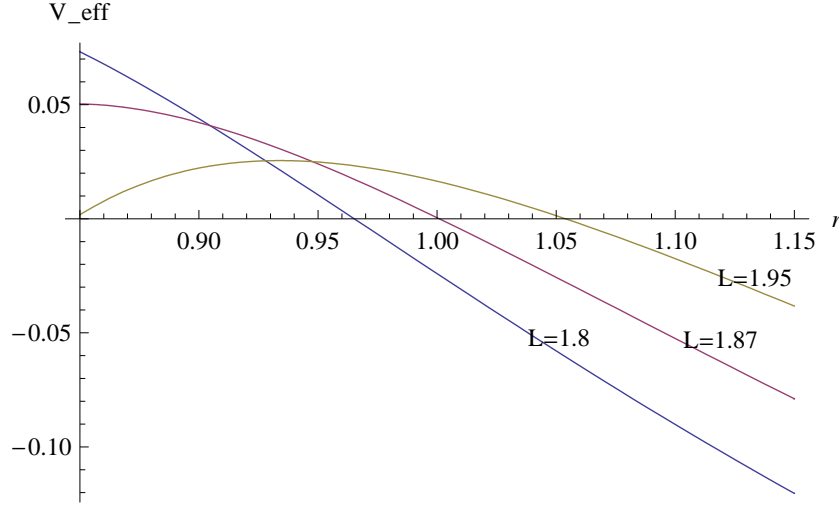


Figure 3. The effective potential is plotted for a particle with $E = 1$. The Kerr parameter is assumed to be $a = 1.005$. It is clear that the angular momentum for the particle to turn back at a radial value $r < 1$ must be lesser than the upper bound $L = 1.87$, which is the critical value for which the particle turns back from $r = 1$. For example, for $L = 1.8$ the particle turns back from $r = 0.96 < 1$, whereas for $L = 1.95$, the particle turns back from $r = 1.05$ which is above the $r = 1$ surface.

reaches $r = 1$ as an outgoing particle, where it interacts with another ingoing particle dropped from infinity at rest.

The proper time required for this process to occur happens to be finite, since both the conditions mentioned in (7) are not satisfied simultaneously anywhere along the geodesic.

When the angular momentum of particles is oriented antiparallel to the spin of the naked singularity with $L < 0$, it can be shown that the simultaneous solution to (11) and (17) does not exist. Thus such particles are not useful towards the purpose of high energy collisions.

The center of mass energy of collision between these two particles is computed using (6). It requires the calculation of the inner product of velocities of the two particles. The velocities of two particles are given by (2),(3),(4) with $E = 1$, and which have appropriate angular momenta as discussed above. The expression for the center of mass energy of collision contains terms that are proportional to $\frac{1}{\Delta}$, those which are independent of Δ , and others with positive powers of Δ . Since the Kerr spin parameter is very close to unity, it follows from (9) that the terms proportional to $\frac{1}{\Delta} \approx \frac{1}{2\epsilon}$ would make a dominant contribution, and the other terms can be neglected being insignificantly small as compared to it. The center of mass energy of collision to the leading order is then given by,

$$\lim_{\epsilon \rightarrow 0} E_{c.m.}^2 = 2m^2 \frac{\tilde{T}_1 \tilde{T}_2}{\epsilon} \rightarrow \infty \quad (20)$$

where $\tilde{T}_1 = T_1(r = 1, L_1, E = 1)$, $\tilde{T}_2 = T_2(r = 1, L_2, E = 1)$, and the functions T_1, T_2 are

defined below (3) in Sec 2. We have, $\tilde{T}_1, \tilde{T}_2 \approx O(1)$. Thus we clearly see that the center of mass energy of collision between two particles is arbitrarily large in the limit where the deviation of a Kerr naked singularity from extremality is small.

5. Discussion and open issues

We first note that the consideration of Kerr naked singular geometries is well-motivated by recent theoretical developments in string theory, which suggest by means of specifically worked out examples, that the timelike naked singularities would be naturally resolved. Possible pathological features associated with them like causality violation would be naturally avoided by the high energy modifications to classical general relativity, and predictability would be restored [6]. In such a case, the cosmic censorship conjecture [7] which forbids the existence of the naked singular solutions in nature becomes obsolete. Quantum gravity resolved classical naked singular solutions are then rendered legal and can be used to do calculation as far as one stays sufficiently away from the ultra-high curvature regime.

The results obtained in this paper basically illustrate the mathematical structure of the Kerr geometry. For the situation we described here to be astrophysically relevant, various other issues need to be addressed. We qualitatively discuss some of these points in this section. Rigorous analysis of these questions and all the open issues is beyond the scope of this paper and will be presented elsewhere in future.

The formation of spherically symmetric naked singularities in gravitational collapse has been studied extensively. There are many such collapse models where the violation of cosmic censorship conjecture occurs [8]. On the contrary, the formation of either rotating blackholes or naked singularities from gravitational collapse is not yet very well understood. The formation of both rotating naked singularities and blackholes in gravitational collapse of $2+1$ dimensional shell collapse has been demonstrated recently [9]. It was also shown that the accretion onto compact objects can spin these up to super-spinning configuration. However, this also requires the compact object to have an analogous quadruple moment apart from mass and angular momentum [10]. It has also been suggested that the near extremal Kerr blackhole can be turned into a naked singularity by throwing in test particles [11], although it is a matter of debate and investigation whether the results would survive after the self-force and backreaction have been taken into account.

The Kerr naked singular solution is not the unique vacuum, asymptotically flat, axially symmetric solution to the Einstein equations, the most general solution being the Tomimatsu-Sato geometries [12]. We analyzed here the simplest subcase, namely the Kerr naked singular solution. We are currently investigating whether the high energy collisions can also take place in the spacetimes with analogous higher multipole moments. Yet another variant of a Kerr naked singular solution is the non-vacuum, axially symmetric, asymptotically flat solution with the massless scalar field as a matter source [13]. We have verified that the high energy collisions do take place in this

geometry admitting rotating naked singularities. Thus it might be reasonable to expect that the results we presented here would carry over to geometries other than the Kerr naked singularity.

We carried out the analysis here under the assumption that the colliding particles are test particles and followed a geodesic motion on the background Kerr geometry. However, in the full calculation, the backreaction and gravitational radiation emitted by infalling particles must be taken into account. In the blackhole case, one of the colliding particle follows a whirl orbit. It is an orbit which asymptotes to the horizon and the particle circles around the horizon many times. Such a particle has a fine-tuned value of the angular momentum. This particle emits a large amount of gravitational radiation and its orbit suffers a severe deviation, thereby reducing the center of mass energy by a large amount [16]. In the process we described here, fine-tuning of the angular momentum is avoided. Thus the gravitational radiation emitted is significantly reduced, since the particle trajectory is not a whirl orbit. Although it is expected that the particle would be deviated from a geodesic motion, since all we need is an ingoing and outgoing particle colliding around $r = 1$, it is expected that there would be high energy collisions. Secondly, if one assumes that the particles would be accreted in the form of quasi-spherical shell then the gravitational radiation per particle would be much lesser than the gravitational radiation emitted by a single infalling particle. The issue of backreaction is as such difficult to deal with here in the absence of spherical symmetry. The full general relativistic calculation, taking into account the backreaction, was carried out in [17] and it was shown that the center of mass energy of collision between the shells around the extremal Reissner-Nordström blackhole turns out to be finite, as opposed to the unbound center of mass energy, when the test shell approximation is used. Contrary to the above result, we have shown recently that in the case of naked singular Reissner-Nordström spacetime the center of mass energy of collision turns out to be unbound even when the exact calculation is carried out taking into account the backreaction [18].

We also note that the particles released from rest at infinity are highly blueshifted as they reach $r = 1$, where they participate in the high energy collisions with extremely large center of mass energy. The high energy particles produced in the collisions will be highly redshifted when they reach out infinity. But there is an overall compensation of blueshift and redshift so that the particles reaching infinity carry energy that is comparable to the mass of the infalling particles. This can also be argued in the following way. The conservation of energy-momentum in the collision implies that the total conserved energy of the colliding particles would be same as the total conserved energy of the collision products. Thus the conserved energy of the particles produced in the collision escaping to infinity would be of the order of the conserved energy of the colliding particles. The conserved energy is the energy of the particle measured at infinity. Thus the particle produced in the collision escaping to infinity will carry the energy that is comparable to the mass of the colliding particle if it is released from rest at infinity. If the mass of the particle is much lesser than the mass of the colliding particle then the particle produced in the collision will reach infinity with large kinetic

energy.

In general the stability of a given spacetime, including either Kerr blackholes or naked singularities, is an open issue and is still under investigation. It has been demonstrated recently that the Kerr naked singular solution admits an instability [14]. We note that Kerr blackholes also could admit certain instabilities [11],[15]. These are different kinds of instabilities associated with the Kerr solution in standard general relativity and also for the $F(R)$ gravity theories. Recently yet another instability is under much discussion, which is that associated with the near extremal blackholes, in the case when the blackhole absorbs charged or rotating particles, possibly turning it into a naked singularity as we discussed above. These instabilities could play a vital role in particle acceleration mechanism associated with the near extremal blackholes. Despite that the extremal blackholes have been extensively studied from the perspective of particle acceleration mechanism in past couple of years.

We note that more careful further analysis is needed to investigate whether or not, and under what circumstances the process we described here will be stable when the back-reaction effects and the gravitational radiation emitted by particles is taken into account. Since this issue is complex one to deal with in Kerr geometry, we have done an exact analogous calculation taking into account the full backreaction, in the Reissner-Nordström naked singular geometry. It turns out in that case that the Reissner-Nordström geometry is stable and the center of mass energy of collisions can be arbitrarily large. So it might be reasonable to expect that similar results might hold good in Kerr geometry as well, possibly under certain restrictive conditions.

6. Concluding remarks

To summarize, we described a process of high energy collision of particles in the vicinity of near extremal Kerr naked singularities which is generic and requires a finite proper time, unlike in the Kerr blackhole case. The genericity is with respect to the finite range and interval of values of angular momenta that is available to particles that participate in the high energy collisions. This is unlike the blackhole case where extreme finetuning of angular momentum is required.

This is in itself an intriguing and interesting result that it is possible to have collisions with large center of mass energies around the Kerr naked singularities. However, for this phenomenon to be physically relevant, it is important to study and understand issues like the possible processes leading to the formation of Kerr naked singularities and the deviation of the colliding particles from geodesic motion due to the gravitational radiation and also the backreaction effects.

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